Journal of Sound and Vibration (2002) **251**(3), 383–394
doi:10.1006/jsvi.2001.3999, available online at http://www.idealibrary.com on IDEAL®

OPTIMAL CONTROL METHOD WITH TIME DELAY IN CONTROL

G. CAI AND J. HUANG

Department of Engineering Mechanics, Shanghai Jiaotong University, Shanghai 200030, *People's Republic of China. E-mail*: *caigp@263.net*

(*Received* 1 *September* 2000, *and in final form* 11 *July* 2001)

Optimal control method for active vibration control of linear time-delay systems is investigated in this paper. In terms of two cases that time delay is integer and non-integer times of sampling period, motion equation with time delay is transformed as standard discrete forms which contain no time delay by using zero order holder respectively. Discrete quadratic function is used as objective function in design of controller to guarantee good control efficiency on sampling points. In every step of computation of the deduced controller, it contains not only current step of state feedback but also linear combination of some former steps of control. Because the controller is deduced directly from time-delay differential equation, system stability can be guaranteed easily, thus this method is generally applicable to ordinary control systems. The performance of the control method proposed and system stability when using this method are all demonstrated by numerical simulation results. Simulation results demonstrate that the presented method is a viable and attractive control strategy for applications to active vibration control. Instability in responses occurs possibly if the systems with time delay are controlled using controller designed in case of no time delay.

1. INTRODUCTION

In recent years, the technique of active vibration control has been developing rapidly, and many active control methods have been used in practical engineering [1–3]. Results obtained both in laboratory demonstrations and in actual measurements for practical engineering show that vibration suppression by active control is emerging as a powerful technique to improve the performance of structures against earthquakes, wind and other dynamic excitations [3–6]. Meanwhile, many problems that limit this technique towards large-scale practical application have been found [7]. Time delay is one of these problems. For example, in active control of large structures, because order of magnitude of control force is often required to be the same as weight of the controlled structures, servohydraulic actuators are often used as control-force delivery devices. But time delay exists obviously in the servohydraulic actuators and it results in unsynchronized control force applied to the structures.

Time delay exists in active control systems inevitably. Time delay can be divided into two classes on the whole. One is in measurement of system variables and calculation for required control force, including physical properties of equipment used in the system or signal transmission. The other is in control for actuators to build up the required control force. The time delay is often omitted for convenience in theoretical analysis and control design before. But even small time delay, it can lead the actuators inputting energy to the structures when no energy is required by the structures, and this can cause degradation in control efficiency or even render the structures unstable [8].

0022-460X/02/130383 + 12 $35.00/0

Optimal control method for vibration control of linear systems with time delay in control is investigated in this paper. Zero order holder is assumed to be used between the controlled structure and controller, thus the time-delay differential equation can be transformed as standard discrete form which contains no time delay. Because the controller is obtained directly from the time-delay differential equation, system stability can be guaranteed easily. Numerical example for a three-story building model is carried out to demonstrate the efficiency of the proposed control method at the end of this paper.

2. EQUATION OF MOTION

Consider a linear structure modelled by an n-degree-of-freedom lumped mass–spring–dashpot system. The matrix equation of motion of the structural system is written as

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{H}\mathbf{U}(t-\lambda) + \mathbf{P}(t), \tag{1}$$

where $\mathbf{X} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ = an n-dimensional vector of displacement; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} = (n \times n)$ mass, damping and stiffness matrices, respectively; $\mathbf{H}$ = a $(n \times r)$ matrix denoting the location of controllers. $\mathbf{U}(t-\lambda)$ = an r-dimensional vector of controllers, in which λ is the time delay. $\mathbf{P}(t)$ is the external excitation.

The time delay λ can be written as follows:

$$\lambda = lT - \bar{m}, \tag{2}$$

where $l > 0$ is any integer; $0 \leqslant \bar{m} < T$; T is the sampling period.

In the state-space representation, equation (1) becomes

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{B}\mathbf{U}(t-\lambda) + \bar{\mathbf{P}}(t), \tag{3}$$

where

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{X}(t) \\ \dot{\mathbf{X}}(t) \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{H} \end{bmatrix}, \quad \bar{\mathbf{P}}(t) = \begin{bmatrix} \mathbf{0} \\ \mathbf{P}(t) \end{bmatrix}.$$

3. DISCRETIZATION AND STANDARDIZATION FOR MOTION EQUATION

The analytical solution of equation (3) can be written as [9, 10]

$$\mathbf{Z}(t) = \mathrm{e}^{\mathbf{A}(t-t_0)}\mathbf{Z}(t_0) + \int_{t_0}^{t} \mathrm{e}^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{U}(\tau-\lambda)\,\mathrm{d}\tau + \int_{t_0}^{t} \mathrm{e}^{\mathbf{A}(t-\tau)}\bar{\mathbf{P}}(\tau)\,\mathrm{d}\tau. \tag{4}$$

Zero order holder is assumed being used in the structure, i.e.,

$$\mathbf{U}(t) = \mathbf{U}(k), \quad kT \leqslant t < (k+1)T, \tag{5}$$

where k means the kth step of control. $\mathbf{U}(k)$ denotes $\mathbf{U}(kT)$ in fact. This kind of denotation is used below for simplicity in expression. Equation (5) means that the actuators in the structure exert constant control forces on the structure during two adjoining sampling points.

Let $t_0 = kT$ and $t = (k+1)T$, equation (4) becomes

$$\mathbf{Z}(k+1) = \mathrm{e}^{\mathbf{A}T}\mathbf{Z}(k) + \int_{kT}^{(k+1)T} \mathrm{e}^{\mathbf{A}(kT+T-\tau)}\mathbf{B}\mathbf{U}(\tau-\lambda)\,\mathrm{d}\tau + \int_{kT}^{(k+1)T} \mathrm{e}^{\mathbf{A}(kT+T-\tau)}\bar{\mathbf{P}}(\tau)\,\mathrm{d}\tau. \tag{6}$$

Making variable substitution $\eta = (k+1)T - \tau$, equation (6) becomes

$$\mathbf{Z}(k+1) = \mathrm{e}^{\mathbf{A}T}\mathbf{Z}(k) + \int_0^T \mathrm{e}^{\mathbf{A}\eta}\mathbf{B}\mathbf{U}(kT+T-lT+\bar{m}-\eta)\,\mathrm{d}\eta + \int_0^T \mathrm{e}^{\mathbf{A}\eta}\bar{\mathbf{P}}(kT+T-\eta)\,\mathrm{d}\eta. \tag{7}$$

Through equation (7), equation (3) can be transformed into standard discrete form in terms of two cases that the time delay, λ, is integer and non-integer times of the sampling period, T (i.e., $\bar{m} = 0$ and $\bar{m} \neq 0$), respectively, as follows [10].

1. $\bar{m} = 0$

When $\bar{m} = 0$, in consideration of equation (5), equation (7) can be written as

$$\begin{aligned}\mathbf{Z}(k+1) &= \mathrm{e}^{\mathbf{A}T}\mathbf{Z}(k) + \int_0^T \mathrm{e}^{\mathbf{A}\eta}\,\mathrm{d}\eta\,\mathbf{B}\mathbf{U}(k-l) + \int_0^T \mathrm{e}^{\mathbf{A}\eta}\bar{\mathbf{P}}(kT+T-\eta)\,\mathrm{d}\eta \\ &= \mathbf{F}\mathbf{Z}(k) + \mathbf{G}\mathbf{U}(k-l) + \bar{\mathbf{P}}(k),\end{aligned} \tag{8}$$

where

$$\mathbf{F} = \mathrm{e}^{\mathbf{A}T}, \quad \mathbf{G} = \int_0^T \mathrm{e}^{\mathbf{A}t}\,\mathrm{d}t\,\mathbf{B}, \quad \bar{\mathbf{P}}(k) = \int_0^T \mathrm{e}^{\mathbf{A}\eta}\bar{\mathbf{P}}(kT+T-\eta)\,\mathrm{d}\eta. \tag{9}$$

Equation (8) is namely the discrete form of the continuous state equation given by equation (3) when $\bar{m} = 0$. For design of optimal controller, equation (8) should be further changed into standard discrete form. For this purpose, let

$$\begin{aligned}\mathbf{Z}_{n+1}(k) &= \mathbf{U}(k-l), \\ \mathbf{Z}_{n+2}(k) &= \mathbf{U}(k-l+1), \\ &\vdots \\ \mathbf{Z}_{n+l}(k) &= \mathbf{U}(k-1)\end{aligned} \tag{10}$$

and let

$$\bar{\mathbf{Z}}(k) = [\mathbf{Z}^{\mathrm{T}}(k), \mathbf{Z}_{n+1}^{\mathrm{T}}(k), \ldots, \mathbf{Z}_{\mathrm{n}+l}^{\mathrm{T}}(k)]^{\mathrm{T}}. \tag{11}$$

Thus, equation (8) can be written as a standard form as follows:

$$\bar{\mathbf{Z}}(k+1) = \bar{\mathbf{F}}\bar{\mathbf{Z}}(k) + \bar{\mathbf{G}}\mathbf{U}(k) + \tilde{\mathbf{P}}(k) \tag{12}$$

in which

$$\bar{\mathbf{F}} = \begin{bmatrix} \mathbf{F} & \mathbf{G} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \dots & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{G}} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \tilde{\mathbf{P}}(k) = \begin{bmatrix} \bar{\mathbf{P}}(k) \\ \vdots \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{13}$$

Iterative algorithms of **F** and **G** given in equation (9) are displayed in Appendix A.

2. $\bar{m} \neq 0$

When $\bar{m} \neq 0$, equation (7) can be written as

$$\begin{aligned} \mathbf{Z}(k+1) &= e^{\mathbf{A}T}\mathbf{Z}(k) + \int_0^{\bar{m}} e^{\mathbf{A}\eta}\, d\eta\, \mathbf{B}\mathbf{U}(k-l+1) + \int_{\bar{m}}^{T} e^{\mathbf{A}\eta} d\eta\, \mathbf{B}\mathbf{U}(k-l) \\ &\quad + \int_0^T e^{\mathbf{A}\eta}\bar{\mathbf{P}}(kT+T-\eta)\, d\eta \\ &= \mathbf{F}\mathbf{Z}(k) + \mathbf{G}_a\mathbf{U}(k-l) + \mathbf{G}_b\mathbf{U}(k-l+1) + \bar{\mathbf{P}}(k) \end{aligned} \tag{14}$$

in which

$$\mathbf{F} = e^{\mathbf{A}T}, \qquad \mathbf{G}_a = \int_{\bar{m}}^{T} e^{\mathbf{A}\eta}\, d\eta\mathbf{B},$$

$$\mathbf{G}_b = \int_0^{\bar{m}} e^{\mathbf{A}\eta} d\eta\mathbf{B}, \quad \bar{\mathbf{P}}(k) = \int_0^T e^{\mathbf{A}\eta}\bar{\mathbf{P}}(kT+T-\eta)\, d\eta. \tag{15}$$

Let

$$\mathbf{F}(t) = e^{\mathbf{A}t}, \quad \mathbf{G}(t) = \int_0^t e^{\mathbf{A}\tau}\, d\tau\mathbf{B} \tag{16}$$

the first three terms in equation (15) can be written as

$$\begin{aligned} \mathbf{F} &= e^{\mathbf{A}T} = \mathbf{F}(T), \\ \mathbf{G}_a &= \int_{\bar{m}}^{T} e^{\mathbf{A}\eta}\, d\eta\mathbf{B} = \int_0^{T-\bar{m}} e^{\mathbf{A}(\bar{m}+\sigma)}\, d\sigma\mathbf{B} \\ &= e^{\mathbf{A}\bar{m}}\int_0^{T-\bar{m}} e^{\mathbf{A}\sigma}\, d\sigma\mathbf{B} = \mathbf{F}(\bar{m})\mathbf{G}(T-\bar{m}), \\ \mathbf{G}_b &= \int_0^{\bar{m}} e^{\mathbf{A}\eta}\, d\eta\mathbf{B} = \mathbf{G}(\bar{m}). \end{aligned} \tag{17}$$

Equation (14) is namely the discrete form of equation (3) when $m \neq 0$. Likewise, equation (14) can be further changed into the following standard discrete form:

$$\bar{\mathbf{Z}}(k+1) = \bar{\mathbf{F}}\bar{\mathbf{Z}}(k) + \bar{\mathbf{G}}\mathbf{U}(k) + \tilde{\mathbf{P}}(k) \tag{18}$$

in which

$$\bar{\mathbf{F}} = \begin{bmatrix} \mathbf{F} & \mathbf{G}_a & \mathbf{G}_b & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \ldots & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \ldots & \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{G}} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \tilde{\mathbf{P}}(k) = \begin{bmatrix} \bar{\mathbf{P}}(k) \\ \vdots \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{19}$$

4. OPTIMAL CONTROL METHOD WITHOUT TIME DELAY IN CONTROL

It is well known that the linear quadratic regular (LQR) is a classical control method for vibration suppression of linear systems. From references [10, 11], we can design the optimal controller according to the following method when there is no time delay in control. It should be noted that the external excitation $\mathbf{P}(t)$ can be neglected in design of the optimal controller.

For the following linear discrete steady system

$$\mathbf{Z}(k+1) = \mathbf{F}\mathbf{Z}(k) + \mathbf{G}\mathbf{U}(k), \quad \mathbf{Z}(0) = \mathbf{Z}_0, \tag{20}$$

the performance index is given by

$$J = \sum_{k=0}^{\infty} [\mathbf{Z}^{\mathrm{T}}(k)\mathbf{Q}\mathbf{Z}(k) + \mathbf{U}^{\mathrm{T}}(k)\mathbf{R}\mathbf{U}(k)] \tag{21}$$

in which $\mathbf{Q}$ is positive-semidefinite symmetric matrix; $\mathbf{R}$ is positive-definite symmetric matrix. Assuming that $\mathbf{S}(k)$ is the solution of the following discrete Riccati difference equation:

$$\mathbf{S}(k) = \mathbf{F}^{\mathrm{T}}\{\mathbf{S}(k+1) - \mathbf{S}(k+1)\mathbf{G}[\mathbf{R} + \mathbf{G}^{\mathrm{T}}\mathbf{S}(k+1)\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{S}(k+1)\}\mathbf{F} + \mathbf{Q},$$

$$\mathbf{S}(N) = \mathbf{Q}_0 \tag{22}$$

hence for arbitrary positive-semidefinite symmetric matrix $\mathbf{Q}_0$, there exists the following result:

$$\mathbf{S} = \lim_{N\to\infty} \mathbf{S}(k, N) = \lim_{N\to-\infty} \mathbf{S}(k, N) \tag{23}$$

and $\mathbf{S}$ is a constant matrix independent of $\mathbf{Q}_0$. Furthermore, $\mathbf{S}$ is a unique positive-definite symmetric matrix determined by the following discrete Riccati algebraic equation:

$$\mathbf{S} = \mathbf{F}^{\mathrm{T}}\{\mathbf{S} - \mathbf{S}\mathbf{G}[\mathbf{R} + \mathbf{G}^{\mathrm{T}}\mathbf{S}\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{S}\}\mathbf{F} + \mathbf{Q} \tag{24}$$

thus, the steady state controller can be written as follows:

$$\mathbf{U}(k) = -\mathbf{L}\mathbf{Z}(k), \qquad \mathbf{L} = [\mathbf{R} + \mathbf{G}^{\mathrm{T}}\mathbf{S}\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{S}\mathbf{F}. \tag{25}$$

This controller can make the performance index J given by equation (21) minimum.

5. OPTIMAL CONTROL METHOD WITH TIME DELAY IN CONTROL

In literatures [10, 12], the optimal design of delay systems are explained in detail. In this section, the discrete optimal controller is discussed when the time delay exists in the control system.

The performance index given by equation (21) can guarantee good control efficiency on sampling points. This performance index is used herein for design of the optimal controller. The controller can be designed in terms of two cases of $\bar{m} = 0$ and $\bar{m} \neq 0$ respectively [10].

1. $\bar{m} = 0$

From the above, the external excitation $\mathbf{P}(t)$ can be neglected in the design of the controller. When $\bar{m} = 0$, the discrete state equation (8) when neglecting $\mathbf{P}(t)$ can be written as

$$\mathbf{Z}(k+1) = \mathbf{F}\mathbf{Z}(k) + \mathbf{G}\mathbf{U}(k-l), \tag{26}$$

where $\mathbf{F}$ and $\mathbf{G}$ are given in equation (9).

Now the question is to design optimal controller by minimizing the objective function J given by equation (21) subjected to the constraint of the discrete state equation (26).

Equation (26) can be changed into the following standard discrete form:

$$\bar{\mathbf{Z}}(k+1) = \bar{\mathbf{F}}\bar{\mathbf{Z}}(k) + \bar{\mathbf{G}}\mathbf{U}(k), \tag{27}$$

where $\bar{\mathbf{Z}}(k)$ is given by equation (11), $\bar{\mathbf{F}}$ and $\bar{\mathbf{G}}$ are given in equation (13).

The performance index J given by (21) can be written as

$$J = \sum_{k=0}^{\infty} [\bar{\mathbf{Z}}^{\mathrm{T}}(k)\hat{\mathbf{Q}}\bar{\mathbf{Z}}(k) + \mathbf{U}^{\mathrm{T}}(k)\hat{\mathbf{R}}\mathbf{U}(k)] \tag{28}$$

in which

$$\hat{\mathbf{Q}} = \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{R}} = \mathbf{R}. \tag{29}$$

So the question is changed to design the optimal controller by minimizing the objective function J given by equation (28) subjected to the constraint of the discrete state equation (27). This optimal controller can be designed by using equations (20)–(25), and expressed as follows:

$$\mathbf{U}(k) = -\mathbf{L}\bar{\mathbf{Z}}(k) = -\mathbf{L}_1\mathbf{Z}(k) - \mathbf{L}_2\mathbf{U}(k-l) - \cdots - \mathbf{L}_{l+1}\mathbf{U}(k-1) \tag{30}$$

in which, $\mathbf{L}_1 \sim \mathbf{L}_{l+1}$ are corresponding dimensional partitioning matrices of matrix $\mathbf{L}$. We can observe from equation (30) that the optimal controller contains not only current step of state feedback term, $\mathbf{Z}(k)$, but also the linear combination of frontal l steps of control magnitudes, $\mathbf{U}(k-l) \sim \mathbf{U}(k-1)$.

2. $\bar{m} \neq 0$

When $\bar{m} \neq 0$, the discrete state equation (14) when neglecting external excitation $\mathbf{P}(t)$ can be written as

$$\mathbf{Z}(k+1) = \mathbf{F}\mathbf{Z}(k) + \mathbf{G}_a\mathbf{U}(k-l) + \mathbf{G}_b\mathbf{U}(k-l+1), \tag{31}$$

where $\mathbf{F}$, $\mathbf{G}_a$ and $\mathbf{G}_b$ are given in equation (15). In the standard form, equation (31) can be written as

$$\bar{\mathbf{Z}}(k+1) = \bar{\mathbf{F}}\bar{\mathbf{Z}}(k) + \bar{\mathbf{G}}\mathbf{U}(k), \tag{32}$$

where $\bar{\mathbf{Z}}(k)$ is given by equation (11), $\bar{\mathbf{F}}$ and $\bar{\mathbf{G}}$ are given in equation (19).

Likewise, the objective function J given by equation (21) can be changed into the form of equation (28). Thus, the optimal controller can be determined by using the same method as given above. This controller has the same expression as equation (30). And equally, this optimal controller contains the linear combination of frontal l steps of control magnitudes apart from current state feedback.

6. NUMERICAL SIMULATION

To demonstrate the applications of the proposed methods and their performance, simulation results for a linear building are presented in this section. A three-story model studied by Yang [13], subjected to horizontal earthquake ground acceleration $\ddot{X}_g(t)$, is considered as shown in Figure 1, in which every story unit is identically constructed. The mass, stiffness and damping coefficient of each story unit are $m_i = 1$ M ton, $k_i = 980$ kN/m, and $c_i = 1{\cdot}407$ kN s/m, respectively ($i = 1$–3). An ABS is installed in the first-story unit. El Centro earthquake (north–south component) scaled to a maximum acceleration of 0·12 $\mathbf{g}$ is used as the input excitation. The earthquake episode is 8 s. Time history of the earthquake is shown in Figure 2. The sampling period is chosen to be $T = 0{\cdot}002$ s. $\mathbf{Q}$ and R in equation (21) are given by $\mathbf{Q} = \mathrm{diag}([10^5, 10^4, 10^3, 1, 1, 1])$ and $R = 1{\cdot}806 \times 10^{-10}$ respectively.

The maximum interstory drifts, x_i, the maximum absolute floor acceleration, $\ddot{x}_i$, without control for the structure are shown in columns 2 and 3 of Table 1.

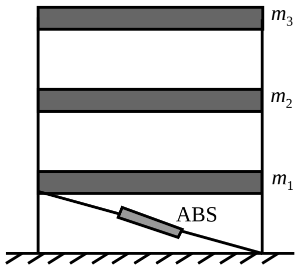

Figure 1. Building model.

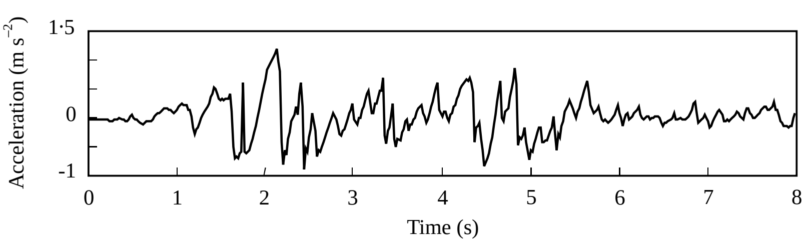

Figure 2. E1 Centro earthquake time history.

TABLE 1

Maximum response quantities (x: *cm*, $\ddot{x}$: *cm/s*^{2})

Story	No control		DLQR $U = 3920$ N		$\bar{m} = 0$				$\bar{m} = 0{\cdot}001$ s				DLQR* $U = 3804$ N	
					$\lambda = 0{\cdot}16\,s$ $U = 4299$ N		$\lambda = 0{\cdot}2$ s $U = 3804$ N		$\lambda = 0{\cdot}159$ s $U = 4297$ N		$\lambda = 0{\cdot}199\,s$ $U = 3817$ N			
(1)	x (2)	$\ddot{x}$ (3)	x (4)	$\ddot{x}$ (5)	x (6)	$\ddot{x}$ (7)	x (8)	$\ddot{x}$ (9)	x (10)	$\ddot{x}$ (11)	x (12)	$\ddot{x}$ (13)	x (14)	$\ddot{x}$ (15)
1	1·37	323	0·10	150	0·49	164	0·51	362	0·49	166	0·51	366	0·10	143
2	1·04	487	0·37	192	0·52	231	0·46	218	0·52	232	0·46	220	0·38	200
3	0·61	599	0·25	246	0·30	297	0·28	278	0·31	300	0·28	279	0·26	257

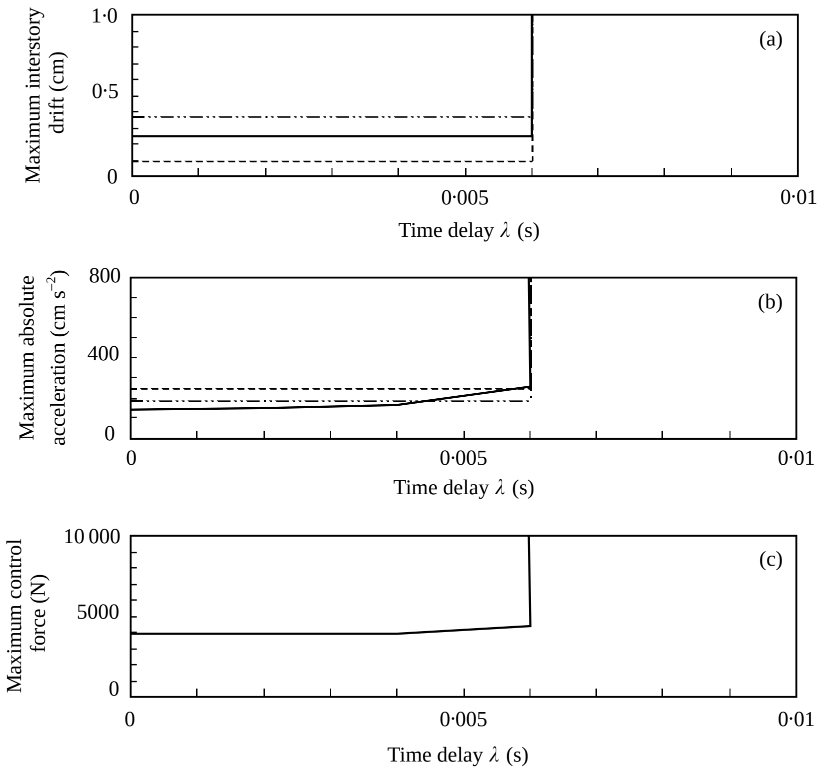

Figure 3. Maximum responses and maximum control-force varied with time delay when the structure with time delay are controlled using the controller designed in case of no time delay. (a) Maximum interstory drifts: - - - - - -, the first-story unit; –··–··–··–··–, the second-story unit; ——, the third-story unit. (b) Maximum absolute acceleration: - - - - - -, the first-story unit; –··–··–··–, the second-story unit; ——, the third-story unit. (c) Maximum control force: ——.

When the controller is designed in case of no time delay, it can be obtained as

$$U(k) = -10^7 \times [1{\cdot}7557x_1(k) - 0{\cdot}1321x_2(k) + 0{\cdot}0777x_3(k)$$

$$+ 0{\cdot}0420\dot{x}_1(k) + 0{\cdot}0210\dot{x}_2(k) + 0{\cdot}0087\dot{x}_3(k)].$$

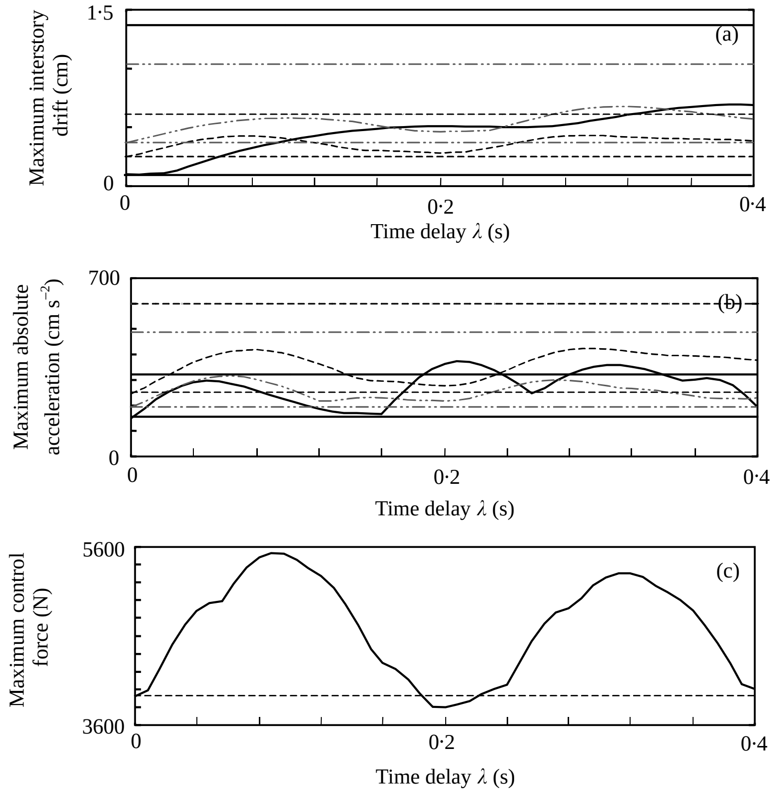

Figure 4. Maximum interstory drifts, maximum absolute acceleration of every story units and maximum control-force varied with λ. (a) Maximum interstory drift: ——, the first-story unit; –··–··–··–, the second-story unit; – – –, the third-story unit. (b) Maximum absolute acceleration: ——, the first-story unit; –··–··–··–··–, the second-story unit; - - - - - -, the third-story unit. (c) Maximum control force: - - - - - -, no time delay; ——, with time delay.

in which $x_1(k)$, $x_2(k)$, $x_3(k)$ are the interstory drifts of every story units of the structure; and $\dot{x}_1(k)$, $\dot{x}_2(k)$, $\dot{x}_3(k)$ are the corresponding velocity quantities. The maximum interstory drifts, x_i, the maximum absolute floor acceleration, $\ddot{x}_i$, of every story units and the maximum required control force, U, are shown in columns 4 and 5 of Table 1, denoted by DLQR.

When the above controller which is designed in case of no time delay is used to control the structure with time delay in control, the curves for the maximum response quantities and the maximum required control force varied with time delay, λ, ($0 \leqslant \lambda \leqslant 0{\cdot}01$ s, i.e., $0 \leqslant l \leqslant 5$) are shown in Figure 3. We can observe from Figure 3 that instability in system responses occurs when λ is very small. In addition, the maximum time delay for stability can be determined to be 0·006 s approximately from Figure 3.

Considering the case of $\bar{m} = 0$, namely that the time delay, λ, is integer times of the sampling period, T. The curves for the maximum response quantities and the maximum required control force varied with $\lambda\,(0 \leqslant \lambda \leqslant 0{\cdot}4\,s$, i.e., $0 \leqslant l \leqslant 200)$ are shown in Figure 4. In Figure 4(a) and 4(b), the upper three beelines denote the case of no control for the structure, the lower three beelines denote the case of the DLQR. In Figure 4(c), the dotted line denotes the case of the DLQR. We can observe from Figure 4 that system stability can be guaranteed independent of the time delay. Every maximum response quantities and the required maximum control forces when $\lambda = 0{\cdot}2$ and 0·16 s are shown in columns of 6–9 of

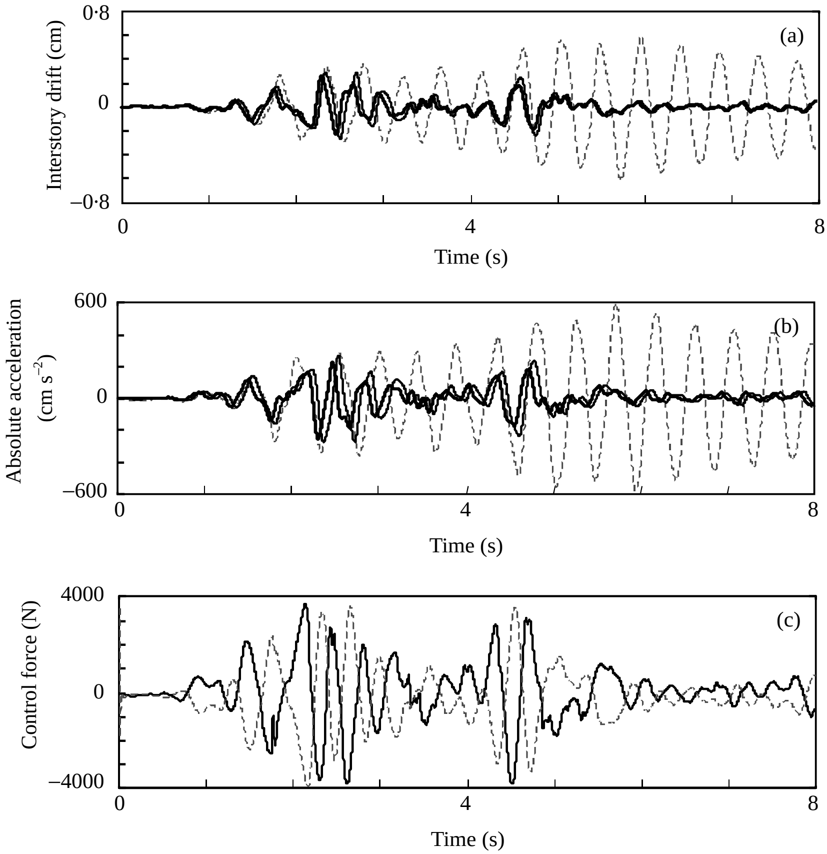

Figure 5. Time histories of responses of the third-story unit and control force. (a) Interstory drift: ------, without control; —— DLQR with no time delay; ———, DLQR with time delay; (b) Absolute acceleration: ------, without control; ——, DLQR with no time delay; ———, DLQR with time delay; (c) Control force: ——, DLQR with no time delay; ------, DLQR with time delay.

Table 1. Time histories of the interstory drift and the absolute floor acceleration of the third-story unit and time history of the control force when $\lambda = 0{\cdot}2$ s are shown in Figure 5. Time histories of responses and the control force when without control and using the DLQR method are given in Figure 5 for comparison. When using the DLQR method, **Q** is still identical to the above, whereas R is adjusted so that the maximum control force is identical to that when $\lambda = 0{\cdot}2$ s, i.e. $U = 3804$ N; the maximum response quantities can be found in columns of 14 and 15 of Table 1, denoted by DLQR*. We can obtain from Figure 5 and Table 1 that the control method proposed is effective in reducing maximum responses of the structure. But its control efficiency is lower in comparison with that when using the DLQR.

Considering the case that λ is non-integer times of T (i.e., $\bar{m} \neq 0$). $\bar{m} = 0{\cdot}001$ s is taken herein. The curves for the maximum response quantities and the maximum required control force varied with λ are similar to Figure 4, and omitted herein. The maximum response quantities and the maximum required control forces, when $\lambda = 0{\cdot}159$ and $0{\cdot}199$ s (i.e., $l = 80$ and 100), are presented in columns 10–13 of Table 1 respectively. From columns (6)–(13), we can observe that the control efficiency when $\lambda = 0{\cdot}2$ and $0{\cdot}16$ s is very close to that when $\lambda = 0{\cdot}199$ and $0{\cdot}159$ s respectively. And this indicates that, when the interval of two time delays is very small, close control efficiencies can be achieved by using controllers designed in terms of $\bar{m} = 0$ and $\bar{m} \neq 0$ respectively.

7. CONCLUSION

Time delay exists in active vibration control inevitably. Control design ignoring may result in a somewhat big error between actual and desired control efficiencies, or even render the structures unstable. Thus, time delay is one of the problems that need a serious attention.

Optimal control method for linear systems with time delay in control is investigated in this paper. Research results demonstrate that good control efficiency can be achieved by using the proposed control method; system stability can be guaranteed by this method. Instability in responses occurs possibly if the systems with time delay are controlled using controller designed in case of no time delay.

REFERENCES

1. S. P. KING 1988 *Aeronautical Journal* **92**, 247–263. Active control of structural response.
2. A. M. MCDONALD, S. J. ELLIOTT and M. A. STOKES 1991 *Proceedings of the International Symposium on Active Control of Sound and Vibration*, 147–156. Active noise and vibration control within the automobile.
3. G. W. HOUSNER, L. A. BERGMAN, T. K. CAUGHEY, A. G. CHASSIAKOS, R. O. CLAUS, S. F. MASRI, R. E. SKELTON, T. T. SOONG, B. F. SPENCER and J. T. P. YAO 1997 *Journal of Engineering Mechanics, American Society of Civil Engineers* **123**, 897–971. Structure control: past, present, and future.
4. Z. WU and T. T. SOONG 1996 *Journal of Engineering Mechanics, American Society of Civil Engineers* **122**, 771–777. Modified bang–bang control law for structural control implementation.
5. J. N. YANG, J. C. WU, A. M. REINHORN, M. RILEY, W. E. SCHMITENDORF and F. JABBARI 1996 *Journal of Structural Engineering, American Society of Civil Engineers* **122**, 69–75. Experimental verifications of H_∞ and sliding mode control for seismically excited buildings.
6. L. L. CHUNG and M. ABDEL-ROHMAN 1988 *Journal of Engineering Mechanics, American Society of Civil Engineers* **114**, 241–256. Experiments on active control of seismic structures.
7. Z. GU, K. MA and W. CHEN 1997 *Active Control of Vibration*. Beijing, China: Defense Industry Press.
8. H. HU 1997 *Chinese Journal of Vibration Engineering* **10**, 273–279. On dynamics in vibration control with time delay.
9. S. LI and L. WEN 1987 *Functional Differential Equation*. Changsha; China: Hunan Science Press.
10. Z. SUN 1989 *Theory and Application of Computer Control*. Beijing, China: Tsinghua University Press.
11. H. KWAKERNAAK and R. SIVAN 1972 *Linear Optimal Control Systems*. New York: Wiley-Interscience.
12. V. KOLMANOVSKII and A. MYSHKIS 1999 *Introduction to the Theory and Applications of Functional Differential Equations*. Dordrecht: Kluwer Academic Publishers.
13. J. N. YANG, J. C. WU and A. K. AGRAWAL 1995 *Journal of Engineering Mechanics, American Society of Civil Engineers* **121**, 1386–1390. Sliding mode control for seismically excited linear structures.

APPENDIX A. ITERATIVE ALGORITHMS OF F AND G

F given in equation (9) can be expanded as

$$\mathbf{F} = \mathrm{e}^{\mathbf{A}T} = \mathbf{I} + \mathbf{A}T + \frac{\mathbf{A}^2 T^2}{2!} + \cdots = \sum_{k=0}^{\infty} \frac{\mathbf{A}^k T^k}{k!}. \tag{A1}$$

G given in equation (9) can be written as

$$\mathbf{G} = \int_0^T \mathrm{e}^{\mathbf{A}t}\,\mathrm{d}t\mathbf{B} = \sum_{k=0}^{\infty} \frac{\mathbf{A}^k T^{k+1}}{(k+1)!}\mathbf{B} = \sum_{k=1}^{\infty} \frac{\mathbf{A}^{k-1} T^k}{k!}\mathbf{B}. \tag{A2}$$

The Euclidean norm of equation (A1) can be written as

$$\|\mathbf{F}\| = \|\mathrm{e}^{\mathbf{A}T}\| = \left\|\sum_{k=0}^{\infty} \frac{(\mathbf{A}T)^k}{k!}\right\| \leqslant \sum_{k=0}^{\infty} \frac{\|\mathbf{A}T\|^k}{k!} = \mathrm{e}^{\|AT\|}. \tag{A3}$$

Because $\mathrm{e}^{\|\mathbf{A}T\|}$ is a scalar, it is bounded as far as $\|\mathbf{A}T\|$ is bounded. Thus the series given by equation (A1) is bounded. Likewise, the series given by equation (A2) is bounded. If **A** is non-singular, **G** given by equation (A2) can be written as

$$\mathbf{G} = \mathbf{A}^{-1}(\mathrm{e}^{\mathbf{A}T} - \mathbf{I})\mathbf{B} = \mathbf{A}^{-1}(\mathbf{F} - \mathbf{I})\mathbf{B}. \tag{A4}$$

Thus, **F** and **G** can be calculated in accordance with equations (A1) and (A4) when **A** is non-singular.

If **A** is singular, equation (A4) cannot be used for **G**. Here we can calculate **F** and **G** according to the following method.

Let

$$\mathbf{G}_1 = \int_0^T \mathrm{e}^{\mathbf{A}t}\,\mathrm{d}t = \sum_{k=1}^{\infty} \frac{\mathbf{A}^{k-1} T^k}{k!} \tag{A5}$$

so we have

$$\mathbf{G} = \mathbf{G}_1\mathbf{B} \tag{A6}$$

in accordance with equation (A1), we can obtain

$$\mathbf{F} = \sum_{k=0}^{\infty} \frac{\mathbf{A}^k T^k}{k!} = \mathbf{I} + \sum_{k=1}^{\infty} \frac{\mathbf{A}^{k-1} T^k}{k!}\mathbf{A} = \mathbf{I} + \mathbf{G}_1\mathbf{A}. \tag{A7}$$

Thus, the algorithms of **F** and **G** when **A** is singular can be summed up as follows:

$$\mathbf{G}_1 = \sum_{k=1}^{\infty} \mathbf{G}_1(k), \qquad \mathbf{G}_1(k) = \frac{\mathbf{A}T}{k}\mathbf{G}_1(k-1),$$

$$\mathbf{G}_1(1) = T\mathbf{I}, \qquad \mathbf{G} = \mathbf{G}_1\mathbf{B}, \qquad \mathbf{F} = \mathbf{I} + \mathbf{G}_1\mathbf{A}. \tag{A8}$$